MULTIMEDIA UNIVERSITY®

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

EEL1176 FIELD THEORY

(All Sections / Groups)

13 MARCH 2020
9:00 A.M–11:00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 8 pages including thiscover page with 4 questions only. Three appendices are provided at the end of the question paper.

2. Attempt all four questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the answer booklet provided.

Question 1

(a) Given $\vec{A} = -\hat{x} + 2\hat{y}$, $\vec{B} = 2\hat{x} - \hat{y} - \hat{z}$, and $\vec{C} = 3\hat{y} + \hat{z}$, compute the following:

(i) $\vec{A} \cdot (\vec{C} \times \vec{B})$

[3 marks]

(ii) $\vec{B} \times (\vec{A} \times \vec{C})$

[3 marks]

(b) Convert Cartesian coordinates (–2,1,–1) to **spherical** coordinates.

[3 marks]

(c) Compute the following:

(i) The **gradient** of $(r \cos \phi)$

[2 marks]

(ii) The **curl** of $(\hat{\phi}\, R \sin\theta \sin\phi)$

[2 marks]

(iii) The **divergence** of $(\hat{\phi}\, R \sin\theta \sin\phi)$

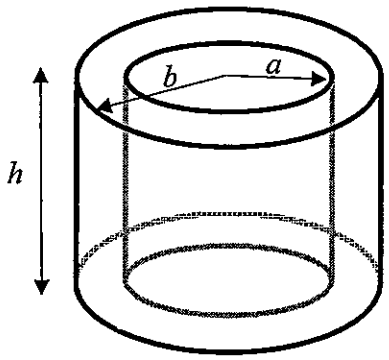
[2 marks]

(d) State the Stoke's theorem and divergence theorem in mathematical terms.

[3 marks]

(e) Given that $\vec{D} = (x^2 + y^2)\hat{y} + 2xy\,\hat{z}$, compute $\oiint_S \vec{D} \cdot ds$ over the closed surface S bounding the hollow cylinder shown in Fig. Q1.

[7 marks]



$a = 2$
$b = 3$
$h = 5$

Fig. Q1

Continued ...

Question 2

(a) Two positive point charges $Q_1 = 3\mu C$ and $Q_2 = -4\mu C$ are placed at (3, 2, 1) and (-4, 0, 6), respectively.

(i) Determine the force, $\vec{F}$ on the charge 20μC at the origin (0, 0, 0).

[6 marks]

(ii) Find the electric field intensity, $\vec{E}$ on the charge 20μC at the origin (0, 0, 0).

[3 marks]

(b) (i) State Gauss law.

[2 marks]

(ii) Write a formula to represent the total charge contained in a given length, *l*.

[2 marks]

(iii) Determine the total charge Q contained in a cylindrical tube of charge oriented along the x-axis. The line charge density is $\rho_l = 3x$, where x is the distance in meters from the bottom end of the tube. The tube length is 10 cm.

[4 marks]

(c) A spherical capacitor is constructed of two concentric spheres of metal with radii of $R_1 = 3$m and $R_2 = 7$m as shown in Fig. Q2. The space between the conductors is filled with two dielectrics, bakelite with relative dielectric constant, $\varepsilon_1 = 5$ occupying one hemisphere while mica with $\varepsilon_2 = 6$ occupying the other hemisphere. Determine the capacitance of the spherical capacitor.

[8 marks]

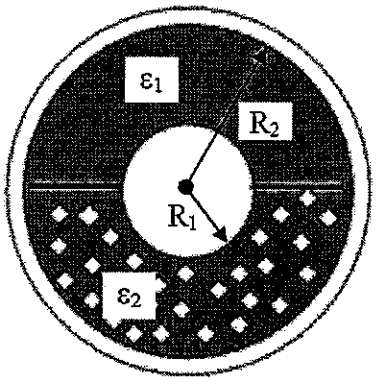


Fig. Q2

Continued ...

Question 3

(a) A 5 nC electric charge with velocity $\vec{u} = 10\,\hat{x}$m/s is moving in the presence of an electric field $\vec{E} = 3\,\hat{y}$V/m and a magnetic field $\vec{B} = 0.5\,\hat{y}$Tesla. Determine the net force experienced by the electric charge.

[5 marks]

(b) Fig. Q3 shows the cross-section of an infinitely long coaxial cable. The inner and outer conductors carry current I in opposite directions. Assume uniform current distribution. Determine the magnetic field intensities $\vec{H}$ for $r \leq a$, $a \leq r \leq b$, $b \leq r \leq c$, and $r \geq c$.

[10 marks]

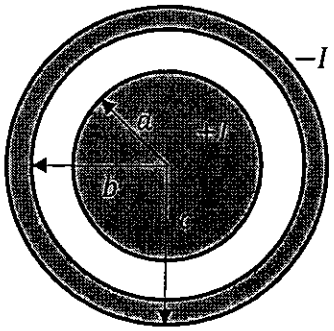


Fig. Q3

(c) A straight solenoid has an iron core ($\mu_r = 1000$) and a cross-sectional area of 100 mm^2. The solenoid is 5 cm in length and it is wound with 100 coil turns. Determine the inductance of the solenoid.

[5 marks]

(d) Two homogeneous, linear and isotropic media have an interface at $x = 0$. Medium 1 is located at $x < 0$ while medium 2 is located at $x > 0$. Medium 1 has relative permeability $\mu_{r1} = 500$ and medium 2 has relative permeability $\mu_{r2} = 1000$. The magnetic field intensity in medium 1 is $\vec{H}_1 = \hat{x} + 2\,\hat{y} - 3\,\hat{z}$(A/m). Assuming no surface current, determine the magnetic field intensity in medium 2, $\vec{H}_2$.

[5 marks]

Continued ...

Question 4

(a) Briefly describe the magnetic materials below and give an example of the corresponding material.

(i) Ferromagnetism.

[3 marks]

(ii) Paramagnetism.

[3 marks]

(iii) Diamagnetism

[3 marks]

(b) A circular cross section ring is made from a magnetic material. The ring has a mean radius of 12 cm and thickness of 1 cm. An air gap of 1 mm length is cut across the ring. The ring is wound with 500 turns of wire carrying a current of 3 A, producing 1 T of flux density in the air gap.
Assuming no fringing effect and no flux leakage, calculate the following:

(i) Magnetic field intensity, H in the magnetic material.

[5 marks]

(ii) Relative permeability, μ_r of the magnetic material.

[5 marks]

(iii) Total reluctance, R_T of the magnetic circuit

[6 marks]

Continued ...

Appendix A: Physical Constants and Vector Basis Transformations

Elementary charge	e	1.60×10^{-19} C
Permittivity of air	ε_o	8.854×10^{-12}F/m
Permeability of air	μ_o	$4\pi \times 10^{-7}$ H/m

Transformation	Vector basis
Cartesian &Cylindrical	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$
	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$
Spherical &Cartesian	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$
	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$
Cylindrical & Spherical	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$
	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$

Continued ...

Appendix B: Gradient, Divergence, Curl, and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}\, r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\phi\right) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}\, R & \hat{\phi}\, R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}\left(A_\phi \sin\theta\right) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}\left(RA_\phi\right)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}\left(RA_\theta\right) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Continued ...

Appendix C: Table of Integrals

$$\int \sin\theta\cos^2\theta d\theta = -\frac{1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\left(a^2-x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2+z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2+z^2}} + \text{constant}$$

$$\int \frac{r\,dr}{\left(r^2+z^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2+z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \text{constant}$$

End of Paper